

Quasi-degenerate neutrino masses with normal and inverted hierarchy

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Abstract

The effect of CP-phases on quasi-degenerate Majorana neutrino (QDN) masses are studied with neutrino mass matrix obeying μ - τ symmetry for normal hierarchy (NH-QD) and inverted hierarchy (IH-QD). We further investigate on (i) the prediction of solar mixing angle below tri-bimaximal value which is consistent with observation, (ii) the prediction on absolute neutrino masses consistent with $0\nu\beta\beta$ decay mass parameter (m_{ee}) and cosmological bound on the sum of the three absolute neutrino masses $\sum_i m_i$. The numerical analysis is carried out through a parameterization of neutrino mass matrices using only two unknown parameters (ϵ, η) within μ - τ symmetry. The results show the validity of QDN mass models in both normal and inverted hierarchical patterns.

Keywords: QDN models, absolute neutrino masses, CP-phases.

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1 Introduction

Since the present neutrino oscillation data [1] on neutrino mass parameters are not sufficient to predict the three absolute neutrino masses in the case of quasi-degenerate neutrino (QDN) mass models [2-8], such absolute mass scale is usually taken as input parameter ranging from 0.1 eV to 0.4 eV in most of the theoretical calculations [9]. As the latest cosmological tightest upper bound on the sum of the three absolute neutrino mass is $\sum_i m_i \leq 0.28$ eV [11], larger value of neutrino mass $m_3 \geq 0.1$ eV in QDN models, has been disfavoured. The upper bound on $m_{ee} \geq 0.2$ eV in $0\nu\beta\beta$ decay [11] also disfavours larger values of neutrino mass eigenvalues with same CP-parity. Some important points for further investigations in QDN models for NH-QD and IH-QD patterns are the searches for QDN models which can accomodate lower values of absolute neutrino masses $m_3 \leq 0.09$ eV, solar mixing angle which is lower than tri-bimaximal mixing (TBM) [12] and effects of CP-phases on neutrino masses. In this paper, we introduce a general classification for QDN models based on their CP-parity patterns and then parameterize the mass matrix within μ - τ symmetry, and finally numerical calculations are carried out.

2 Parameterization of neutrino mass matrix

A general μ - τ symmetric neutrino mass matrix [13,14] with its four unknown independent matrix elements, requires at least four

independent equations for realistic numerical solution,

$$m_{LL} = \begin{pmatrix} m_{11} & m_{12} & m_{12} \\ m_{12} & m_{22} & m_{23} \\ m_{12} & m_{23} & m_{22} \end{pmatrix}. \quad (1)$$

The three mass eigenvalues m_i and solar mixing angle θ_{12} , are given by

$$m_1 = m_{11} - \sqrt{2} \tan \theta_{12} m_{12},$$

$$m_2 = m_{11} + \sqrt{2} \cot \theta_{12} m_{12},$$

$$m_3 = m_{22} - m_{23}.$$

$$\tan 2\theta_{12} = \frac{2\sqrt{2}m_{12}}{m_{11} - m_{22} - m_{23}}. \quad (2)$$

The observed mass-squared differences are calculated as

$$\Delta m_{12}^2 = m_2^2 - m_1^2 > 0, \quad \Delta m_{32}^2 = |m_3^2 - m_2^2|. \quad (3)$$

In the basis where charged lepton mass matrix is diagonal, we have the leptonic mixing matrix, $U_{PMNS} = U$, where

$$U_{PMNS} = \begin{pmatrix} \cos \theta_{12} & \sin \theta_{12} & 0 \\ \frac{\sin \theta_{12}}{\sqrt{2}} & \frac{\cos \theta_{12}}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{\sin \theta_{12}}{\sqrt{2}} & \frac{\cos \theta_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (4)$$

The mass parameters m_{ee} in $0\nu\beta\beta$ decay and the sum of the absolute neutrino masses in WMAP cosmological bound $\sum_i m_i$, are given respectively by,

$$m_{ee} = |m_1 U_{e1}^2 + m_2 U_{e2}^2 + m_3 U_{e3}^2|, m_{cosmos} = m_1 + m_2 + m_3. \quad (5)$$

A general classification for three-fold quasi-degenerate neutrino mass models [13] with respect to Majorana CP-phases in their three mass eigenvalues, is adopted here. Diagonalization of left-handed Majorana neutrino mass matrix m_{LL} in eq.(1) is given by $m_{LL} = U D U^T$, where U is the diagonalising matrix in eq.(4) and $D = \text{Diag}(m_1, m_2 e^{i\alpha}, m_3 e^{i\beta})$ is the diagonal matrix with two unknown Majorana phases (α, β) . In the basis where charged lepton mass matrix is diagonal, the leptonic mixing matrix is given by $U = U_{PMNS}$ [14]. We then adopt the following classification according to their CP-parity patterns in the mass eigenvalues m_i namely Type IA: (+-+) for $D = \text{Diag}(m_1, -m_2, m_3)$; Type IB: (+++) for $D = \text{Diag}(m_1, m_2, m_3)$ and Type-IC: for (++-) for $D = \text{Diag}(m_1, m_2, -m_3)$ respectively. We now introduce the following parameterization for μ - τ symmetric neutrino mass matrices m_{LL} which could satisfy the above classifications [13].

3 Numerical Analysis and Results

For numerical computation of absolute neutrino masses, we take the following observational data:

$$\Delta m_{12}^2 = (m_2^2 - m_1^2) = 7.60 \times 10^{-5} eV^2, \\ |\Delta m_{32}^2| = |m_3^2 - m_2^2| = 2.40 \times 10^{-3} eV^2;$$

and define the following parameters $\phi = \frac{|\Delta m_{23}^2|}{m_3^2}$ and $\psi = \frac{\Delta m_{21}^2}{|\Delta m_{23}^2|}$, where m_3 is the input quantity allowed by the latest cosmological bound. For NH-QD, the other two mass eigenvalues are estimated as, $m_2 = m_3 \sqrt{1 - \phi}$; $m_1 = m_3 \sqrt{1 - \phi(1 + \psi)}$ and for IH-QD as $m_2 = m_3 \sqrt{1 + \phi}$; $m_1 = m_3 \sqrt{1 + \phi(1 - \psi)}$.

For suitable input value of m_3 one can estimate the numerical values of m_1 and m_2 for both NH-QD and IH-QD cases, using the observational values of $|\Delta m_{23}^2|$ and Δm_{21}^2 . Table-1 gives the calculated numerical values for two models namely NH-QD and IH-QD for $|\Delta m_{23}^2| = 7.60 \times 10^{-5} eV^2$ and $\Delta m_{21}^2 = 2.40 \times 10^{-3} eV^2$.

Parameterizations: In the next step we parameterize the mass matrix eq.(1) into three types:

Type IA with $D = \text{Diag}(m_1, -m_2, m_3)$. The mass matrix of this type [13,15] can be parameterized using two parameters (ϵ, η) :

$$m_{LL} = \begin{pmatrix} \epsilon - 2\eta & -c\epsilon & -c\epsilon \\ -c\epsilon & \frac{1}{2} - d\eta & -\frac{1}{2} - \eta \\ -c\epsilon & -\frac{1}{2} - \eta & \frac{1}{2} - d\eta \end{pmatrix} m_3. \quad (6)$$

This predicts the solar mixing angle,

$$\tan^2 \theta_{12} = -\frac{2c\sqrt{2}}{1 + (d-1)\frac{d}{\epsilon}}. \quad (7)$$

when we choose the constant parameters $c=d=1.0$, we get the tri-bimaximal mixings (TBM) $\tan 2\theta_{12} = -2\sqrt{2}$ which leads to $\tan^2 \theta_{12} = 0.50$ and the values of ϵ and η are calculated for both NH-QD and IH-QD cases, by using the values of Table-1 in these two eigenvalue expressions: $m_1 = (2\epsilon - 2\eta)m_3$ and $m_2 = (-\epsilon - 2\eta)m_3$ which are extracted after diagonalization of eq.(6). The results are given in Table-2 for $\tan^2 \theta_{12} = 0.50$. The solar angle can be further lowered by taking the values $c < 1$ and $d < 1$ while using the earlier values of ϵ and η extracted for TBM case. For $\tan^2 \theta_{12} = 0.45$ case the results are shown in Table-3.

Type-IB with $D = \text{Diag}(m_1, m_2, m_3)$: This type [13,15] of quasi-degenerate mass pattern is given by the mass matrix,

$$m_{LL} = \begin{pmatrix} 1 - \epsilon - 2\eta & c\epsilon & c\epsilon \\ c\epsilon & 1 - d\eta & -\eta \\ c\epsilon & -\eta & 1 - d\eta \end{pmatrix} m_3. \quad (8)$$

This predicts the solar mixing angle,

$$\tan 2\theta_{12} = \frac{2c\sqrt{2}}{1 + (1-d)\frac{d}{\epsilon}}. \quad (9)$$

<i>input</i> m_3	<i>calculated</i> ϕ	<i>NH-QD</i>		<i>IH-QD</i>	
		m_1	m_2	m_1	m_2
0.40	0.015	0.39689	0.39699	0.40289	0.40299
0.10	0.24	0.08674	0.08718	0.11104	0.11135
0.08	0.375	0.06264	0.06325	0.09340	0.09380

Table 1: The absolute neutrino masses in eV are estimated from oscillation data (using calculated $\psi = 0.031667$).

which gives the TBM solar mixing angle with the input values $c = 1$ and $d = 1$. When $\epsilon = 0$, $\eta = 0$, this leads to $m_{LL}^{diag} = diag(1, 1, 1)m_3$. Like in Type-IA, here ϵ and η values are computed for NH-QD and IH-QD, by using Table-1 in $m_1 = (1 - 2\epsilon - 2\eta)m_3$ and $m_2 = (1 + \epsilon - 2\eta)m_3$ which are extracted from diagonalization of eq.(8).

Type-IC with $D = \text{Diag}(m_1, m_2, -m_3)$: It is not necessary to treat this model [13] separately as it is similar to Type-IB except with the interchange of two matrix elements (m_{22}) and (m_{23}) in the mass matrix in eq.(10), and this effectively imparts an additional odd CP-parity on the third mass eigenvalue m_3 in Type-IC. Such change does not alter the predictions of Type-IB. Tables 2 and 3 present our numerical results for both $\tan^2\theta_{12} = 0.5$ and 0.45 cases, in all types of QD models (Types-IA, IB, IC). These results are consistent with observational cosmological bound.

4 Conclusion

To conclude, we have studied the effects of Majorana phases on the prediction of absolute neutrino masses in three types of QDN models having both normal and inverted hierarchical patterns within μ - τ symmetry. These predictions are consistent with data on the mass squared difference derived from various oscillation experiments, and from the upper bound on absolute neutrino masses in $0\nu\beta\beta$ decay as well as upper bound of $\sum_i m_i \leq 0.28$ eV from cosmology. The QD models are still far from discrimination and the prediction on solar mixing angle is found to be lower than TBM viz, $\tan^2\theta_{12} = 0.45$ which coincides with the best-fit in the neutrino oscillation data. The result shows the validity of NH-QD and IH-QD models. The results presented in this article are new and have important implications in the discrimination of neutrino mass models.

<i>Different parameters</i>	<i>NH-QD</i>		<i>IH-QD</i>	
	Type-IA	Type-IB	Type-IA	Type-IB
c	1.0	1.0	1.0	1.0
d	1.0	1.0	1.0	1.0
m_3	0.10	0.10	0.08	0.08
ϵ	0.57972	0.0015	0.78004	0.00169
η	0.14602	0.0649	0.19628	-0.08546
m_1 (eV)	0.08674	0.08675	0.09340	0.09340
m_2 (eV)	-0.08717	0.08717	-0.09380	0.09380
m_3 (eV)	0.10	0.10	0.08	0.08
$\sum m_i eV$	0.27	0.274	0.267	0.274
$\Delta m_{21}^2 eV^2$	7.6×10^{-5}	7.6×10^{-5}	7.6×10^{-5}	7.6×10^{-5}
$ \Delta m_{23}^2 eV^2$	2.2×10^{-3}	2.4×10^{-3}	2.4×10^{-3}	2.4×10^{-3}
$\tan^2 \theta_{12}$	0.50	0.50	0.50	0.50
$ m_{ee} $ eV	0.08688	0.0869	0.09354	0.09354

Table 2: Predictions for $\tan \theta_{12} = 0.50$

<i>Different parameters</i>	<i>NH-QD</i>		<i>IH-QD</i>	
	Type-IA	Type-IB	Type-IA	Type-IB
c	0.868	0.945	0.868	0.96
d	1.025	0.998	1.0	1.002
m_3	0.10	0.10	0.08	0.08
ϵ	0.6616	0.00145	0.88762	0.00169
η	0.1655	0.06483	0.22317	-0.08546
m_1 (eV)	0.0876	0.08676	0.09392	0.09341
m_2 (eV)	-0.0880	0.08717	-0.09432	0.09381
m_3 (eV)	0.0996	0.10002	0.08	0.080014
$\sum m_i eV$	0.274	0.274	0.268	0.267
$\Delta m_{21}^2 eV^2$	7.7×10^{-5}	7.3×10^{-5}	7.6×10^{-5}	7.4×10^{-5}
$ \Delta m_{23}^2 eV^2$	2.2×10^{-3}	2.4×10^{-3}	2.4×10^{-3}	2.4×10^{-3}
$\tan^2 \theta_{12}$	0.45	0.45	0.45	0.45
$ m_{ee} $ eV	0.0877	0.08688	0.09403	0.09354

Table 3: Predictions for $\tan \theta_{12} = 0.45$

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